Radiative and leptonic decays of the pseudoscalar charmonium state η_c

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The radiative and leptonic decays of $\eta_c \to \gamma\gamma$ and $\eta_c \to l^+l^-$ are studied. For $\eta_c \to \gamma\gamma$ decay, the second-order electromagnetic tree-level diagram gives the leading contribution. The decay rate of $\eta_c \to \gamma\gamma$ is calculated, the prediction is in good agreement with the experimental data. For $\eta_c \to l^+l^-$, both the tree and loop diagrams are calculated. The analysis shows that the loop contribution dominates, the contribution of tree diagram with Z^0 intermediate state can only modifies the decay rate by less than 1%. The prediction of the branching ratios of $\eta_c \to e^+e^-$ and $\mu^+\mu^-$ are very tiny within the standard model. The smallness of these predictions within the standard model makes the leptonic decays of η_c sensitive to physics beyond the standard model. Measurement of the leptonic decay may give information of new physics.

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The radiative and leptonic decays of $\eta_c \to \gamma\gamma$ and $\eta_c \to l^+l^-$ involve electromagnetic and weak interactions. The decay rates are determined not only by electroweak interaction, but also by strong interaction, which binds the quark-antiquark pair $c\bar{c}$ in η_c together. The decay amplitudes of these decay processes are generally convolutions of the wave function of η_c and the electroweak transition amplitude of $c\bar{c} \to \gamma\gamma$ and/or $c\bar{c} \to l^+l^-$. The decay process of $\eta_c \to \gamma\gamma$ can be used to test the decay constant and the wave function of $c\bar{c}$ in this meson state. The decay rate of $\eta_c \to \gamma\gamma$ has been measured in experiment [1]. Within the framework of the standard model, the leptonic decay $\eta_c \to l^+l^-$ is contributed by fourth-order electromagnetic transition and tree-level weak transition induced by Z^0. The low probability of a fourth-order electromagnetic and weak transition makes the leptonic decays sensitive to hypothetical interactions arising from physics beyond the standard model, such as the existence of a light pseudoscalar Higgs boson in the next-to-minimal supersymmetric standard model [2, 3, 4, 5, 6] or leptoquark bosons that carry both quark and lepton flavors [7, 8, 9, 10], both of which can enhance the leptonic decays of η_c with appropriate values of new physics parameters in these models.

In the literature there have been many theoretical works on the calculations of two-photon decays of pseudoscalar heavy quarkonia η_c, η_b etc., based on relativistic quark model or potential model [11, 12, 13], Bethe-Salpeter equation [14, 15], heavy-quark spin symmetry [16], and Lattice QCD [17]. However the decay rates of leptonic decays of pseudoscalar heavy quarkonia have not been known yet.

In this work the radiative and leptonic decays of $\eta_c \to \gamma\gamma$ and $\eta_c \to l^+l^-$ are studied consistently within the framework of the standard model, by using a method different from those used in the literature. The effective Hamiltonians in quark level for these decays are calculated at first. Then the factorization formula is derived, the decay amplitudes are expressed as convolutions of meson wave function and the hard transition amplitudes, where the wave function is controlled by nonperturbative QCD, for which I use the result calculated in QCD sum rule [18], while the hard transition amplitudes are calculated with perturbation theory. For the process $\eta_c \to l^+l^-$, the loop diagrams are calculated analytically. The infrared divergence is analyzed, the possible information of new physics is also briefly discussed.

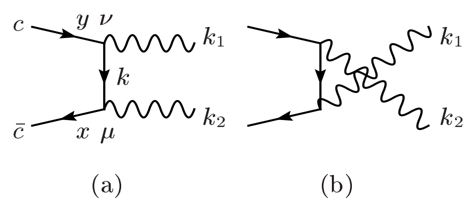

FIG. 1: Diagrams for $\eta_c \to \gamma\gamma$.

The diagram for the decay of $\eta_c \to \gamma\gamma$ is depicted in Fig.1. The transition matrix element relevant to Fig.1(a) can be written in coordinate space as

$$T_1 = \langle\gamma\gamma| \int d^4x d^4y \bar{c}(x) iQ_c e\gamma_\mu \int \frac{d^4k}{(2\pi)^4} \cdot \frac{i}{\not{k} - m_c} e^{-ik\cdot(x-y)} iQ_c e\gamma_\nu c(y) A^\mu(x) A^\nu(y)|\eta_c\rangle, \quad (1)$$

where e is the absolute value of the charge of electron, Q_c the charge of the c quark in unit of e, $A^\mu(x)$ and $A^\nu(y)$ are the electromagnetic fields for the two photons. Contracting the creation operators of the two photons with that in the electromagnetic fields, then the amplitude T_1

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becomes

$$T_1 = \int d^4x d^4y \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)} \tag{2}$$
$$\cdot [iQ_c e\gamma_\mu \frac{i}{\not{k} - m_c} iQ_c e\gamma_\nu]_{\alpha\beta} \epsilon_1^{*\mu} e^{ik_1 x} \epsilon_2^{*\nu} e^{ik_2 y}$$
$$\cdot \langle 0|\bar{c}(x)_\alpha c(y)_\beta|\eta_c\rangle,$$

where α and β are the Dirac spinor indices. It is understood that the repeated indices are summed. In both eqs. (1) and (2), summations over color degrees of freedom and a factor $P\exp\{i\int dz^\mu T^a A^a_\mu(z)\}$ between $\bar{c}(x)$ and $c(y)$ that makes the amplitude gauge-invariant are indicated.

The leading-twist wave function for η_c meson can be defined through the matrix element $\langle 0|\bar{c}(x)_\alpha c(y)_\beta|\eta_c\rangle$ [19, 20], which will be further discussed in Appendix A,

$$\langle 0|\bar{c}(x)_\alpha c(y)_\beta|\eta_c\rangle \tag{3}$$
$$= -\frac{i}{4} f_{\eta_c} \int_0^1 du e^{-i(up\cdot x + \bar{u}p\cdot y)} [\not{p}\gamma_5]_{\beta\alpha} \phi(u,\mu),$$

where $\bar{u} = 1-u$, μ is an energy scale, and f_{η_c} is the decay constant of η_c, which is defined as

$$\langle 0|\bar{c}\gamma_\mu\gamma_5 c|\eta_c\rangle = if_{\eta_c} p_\mu, \tag{4}$$

here p_μ is the four-momentum of the meson.

With the matrix element given in eq.(3), the transition matrix element T_1 becomes

$$T_1 = -\frac{1}{4} f_{\eta_c} Q_c^2 e^2 \int_0^1 du \int d^4x d^4y \int \frac{d^4k}{(2\pi)^4} \tag{5}$$
$$\cdot e^{-ik\cdot(x-y)} e^{ik_1\cdot x} e^{ik_2\cdot y} e^{-i(up\cdot x + \bar{u}p\cdot y)} \phi(u,\mu) \epsilon_1^{*\mu} \epsilon_2^{*\nu}$$
$$\frac{1}{k^2 - m_c^2} Tr[\not{p}\gamma_5\gamma_\mu(\not{k} + m_c)\gamma_\nu].$$

It is not difficult to perform the integration over the coordinates x, y and the momentum k. After these manipulations, one can obtain

$$T_1 = if_{\eta_c} Q_c^2 e^2 \int_0^1 du \frac{\phi(u,\mu)}{(\bar{u}k_1 - uk_2)^2 - m_c^2} \tag{6}$$
$$\cdot \epsilon_{\rho\mu\sigma\nu} k_1^\rho k_2^\sigma \epsilon_1^{*\mu} \epsilon_2^{*\nu} (2\pi)^4 \delta^4(k_1 + k_2 - p).$$

Remove the overall four-momentum conservation factor $(2\pi)^4\delta^4(k_1 + k_2 - p)$, one can obtain the decay amplitude contributed by Fig.1(a)

$$A_1 = if_{\eta_c} Q_c^2 e^2 \int_0^1 du \frac{\phi(u,\mu)}{(\bar{u}k_1 - uk_2)^2 - m_c^2} \tag{7}$$
$$\cdot \epsilon_{\rho\mu\sigma\nu} k_1^\rho k_2^\sigma \epsilon_1^{*\mu} \epsilon_2^{*\nu}.$$

The contribution of Fig.1(b) can be obtained by making the exchange of $k_1 \leftrightarrow k_2$ and $\epsilon_1 \leftrightarrow \epsilon_2$ in A_1

$$A_2 = A_1 \begin{pmatrix} \epsilon_1 \leftrightarrow \epsilon_2 \\ k_1 \leftrightarrow k_2 \end{pmatrix}. \tag{8}$$

Then the total contribution of Fig.1 (a) and (b) is

$$A = A_1 + A_2. \tag{9}$$

Using $k_1^2 = k_2^2 = 0$ and $2k_1\cdot k_2 = m_{\eta_c}^2$, one can simplify the amplitude A to be

$$A = 2if_{\eta_c} Q_c^2 e^2 \int_0^1 du \frac{\phi(u,\mu)}{u\bar{u}m_{\eta_c}^2 + m_c^2} \tag{10}$$
$$\cdot \epsilon_{\rho\sigma\mu\nu} k_1^\rho k_2^\sigma \epsilon_1^{*\mu} \epsilon_2^{*\nu}.$$

For simplicity in the following calculation, one can define a new quantity M as

$$M \equiv 2if_{\eta_c} Q_c^2 e^2 \int_0^1 du \frac{\phi(u,\mu)}{u\bar{u}m_{\eta_c}^2 + m_c^2}, \tag{11}$$

then we can obtain the square of the total amplitude

$$|A|^2 = \frac{1}{2} m_{\eta_c}^4 |M|^2. \tag{12}$$

The decay width can be calculated by the following formula

$$\Gamma(\eta_c \to \gamma\gamma) = \frac{1}{2!}\frac{1}{8\pi}|A|^2 \frac{|\vec{k}|}{m_{\eta_c}^2}, \tag{13}$$

where $\frac{1}{2!}$ is the statistic factor for two identical particles, $\vec{k}$ is the three-momentum of one of the photons in the rest frame of η_c.

Next let us discuss the leptonic decay $\eta_c \to l^+l^-$. In the standard model, the Feynman diagrams for this process include photon-Fermion loop diagrams and Z^0 tree diagram, which are depicted in Fig.2.

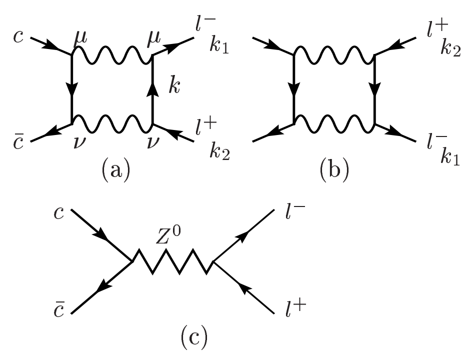


FIG. 2: Diagrams for $\eta_c \to l^+l^-$. (a) and (b) QED contribution where two virtual photons as intermediate state, (c) weak Z^0 contribution.

The effective Hamiltonian describing the transition of $c\bar{c} \to l^+l^-$ can be calculated with the help of the Feynman diagram in Fig.2. The contribution of Fig.2 (a) is

calculated to be

$$H_1 = Q_c^2 e^4 \int \frac{d^4k}{(2\pi)^4} \cdot \frac{\bar{l}\gamma^\mu(\not{k}+m_l)\gamma^\nu l\ \bar{c}\gamma_\nu(\not{k}-\not{k}_1+\not{p}_c+m_c)\gamma_\mu c}{\{(k^2-m_l^2+i\epsilon)[(k-k_1)^2+i\epsilon][(k+k_2)^2+i\epsilon] \times[(k-k_1+p_c)^2-m_c^2+i\epsilon]\}}, \tag{14}$$

where m_l is the lepton mass, p_c the four-momentum of c quark. The quark fields c and $\bar{c}$ are in the momentum space. They are related to the field operators in the coordinate space by

$$\bar{c} = \int d^4x \bar{c}(x) e^{ip_{\bar{c}}\cdot x}, \tag{15}$$

$$c = \int d^4y c(y) e^{ip_c\cdot y}. \tag{16}$$

Then the effective Hamiltonian H_1 can be reexpressed in terms of quark fields in the coordinate space

$$H_1 = \int d^4x e^{ip_{\bar{c}}\cdot x}\int d^4y e^{ip_c\cdot y} Q_c^2 e^4 \int \frac{d^4k}{(2\pi)^4} \times \frac{\bar{l}\gamma^\mu(\not{k}+m_l)\gamma^\nu l\ \bar{c}(x)\gamma_\nu(\not{k}-\not{k}_1+\not{p}_c+m_c)\gamma_\mu c(y)}{a_k}, \tag{17}$$

where a_k is defined as

$$a_k \equiv (k^2-m_l^2+i\epsilon)[(k-k_1)^2+i\epsilon] \times[(k+k_2)^2+i\epsilon][(k-k_1+p_c)^2-m_c^2+i\epsilon]. \tag{18}$$

With the effective Hamiltonian H_1, the amplitude contributed by Fig.2 (a) is

$$\begin{aligned} A_1 &= \int \frac{d^4p_c}{(2\pi)^4}\frac{d^4p_{\bar{c}}}{(2\pi)^4}\langle l^+l^-|H_1|\eta_c\rangle \\ &= Q_c^2e^4\int \frac{d^4p_c}{(2\pi)^4}\frac{d^4p_{\bar{c}}}{(2\pi)^4}\int d^4x e^{ip_{\bar{c}}\cdot x}\int d^4y e^{ip_c\cdot y} \\ &\quad \cdot \int \frac{d^4k}{(2\pi)^4}\bar{u}(k_1)\gamma^\mu(\not{k}+m_l)\gamma^\nu v(k_2) \\ &\quad \times\langle 0|\bar{c}(x)\gamma_\nu(\not{k}-\not{k}_1+\not{p}_c+m_c)\gamma_\mu c(y)|\eta_c\rangle\frac{1}{a_k}, \end{aligned} \tag{19}$$

The matrix element in the above equation can be treated as

$$\begin{aligned} &\langle 0|\bar{c}(x)\gamma_\nu(\not{k}-\not{k}_1+\not{p}_c+m_c)\gamma_\mu c(y)|\eta_c\rangle \\ &= \langle 0|\bar{c}(x)_\alpha c(y)_\beta|\eta_c\rangle[\gamma_\nu(\not{k}-\not{k}_1+\not{p}_c+m_c)\gamma_\mu]_{\alpha\beta}. \end{aligned} \tag{20}$$

With the help of eq.(20) and the wave function of η_c defined in eq.(3), the decay amplitude A_1 can be expressed as

$$\begin{aligned} A_1 &= Q_c^2e^4\int\frac{d^4k}{(2\pi)^4}\bar{u}(k_1)\gamma^\mu(\not{k}+m_l)\gamma^\nu v(k_2) \\ &\times \frac{-i}{4}f_{\eta_c}\int_0^1\phi(u,\mu)\mathrm{Tr}[\not{p}\gamma_5\gamma_\nu(\not{k}-\not{k}_1+\bar{u}\ \not{p} \\ &+m_c)\gamma_\mu]\frac{1}{a_k}. \end{aligned} \tag{21}$$

Perform the trace operation and using the identity of the gamma matrices

$$\gamma_\alpha\gamma_\beta\gamma_\lambda = g_{\alpha\beta}\gamma_\lambda + g_{\beta\lambda}\gamma_\alpha - g_{\alpha\lambda}\gamma_\beta + i\epsilon_{\mu\alpha\beta\lambda}\gamma^\mu\gamma_5, \tag{22}$$

the amplitude A_1 can be further calculated to be

$$\begin{aligned} A_1 &= iQ_c^2e^4 f_{\eta_c}\int_0^1 du\phi(u,\mu)[\bar{u}(k_1)\gamma^\alpha\gamma_5 \\ &\cdot v(k_2)a_{1\alpha} - m_l\bar{u}(k_1)\sigma^{\mu\nu}v(k_2)\epsilon_{\mu\nu\rho\sigma}p^\rho a_2^\sigma], \end{aligned} \tag{23}$$

where

$$a_{1\alpha} = 2\int\frac{d^4k}{(2\pi)^4} \times\frac{p_\alpha k\cdot(k-k_1+\bar{u}p)-(k-k_1+\bar{u}p)_\alpha p\cdot k}{a_k}, \tag{24}$$

$$a_2^\sigma = \int\frac{d^4k}{(2\pi)^4}\frac{(k-k_1+\bar{u}p)^\sigma}{a_k}. \tag{25}$$

Next one needs to perform the loop integrations in the coefficients $a_{1\alpha}$ and a_2^σ.

Using $p = k_1 + k_2$ and a few steps of algebra manipulation, one can cancel one propagator in the coefficient $a_{1\alpha}$, then the four-point loop integration of $a_{1\alpha}$ can be reduced to a sum of a few three-point loop integrals

$$\begin{aligned} a_{1\alpha} = \int\frac{d^4k}{(2\pi)^4}\Big\{\ p_\alpha\times\Big[&\frac{1}{[(k-k_1)^2+i\epsilon][(k+k_2)^2+i\epsilon][(k-k_1+\bar{u}p)^2-m_c^2+i\epsilon]} \\ &+\frac{u}{(k^2-m_l^2+i\epsilon)[(k+k_2)^2+i\epsilon][(k-k_1+\bar{u}p)^2-m_c^2+i\epsilon]}+\frac{\bar{u}}{(k^2-m_l^2+i\epsilon)[(k-k_1)^2+i\epsilon][(k-k_1+\bar{u}p)^2-m_c^2+i\epsilon]}\Big] \\ &-\frac{(k-uk_1+\bar{u}k_2)_\alpha}{(k^2-m_l^2+i\epsilon)[(k-k_1)^2+i\epsilon][(k-k_1+\bar{u}p)^2-m_c^2+i\epsilon]}+\frac{(k-uk_1+\bar{u}k_2)_\alpha}{(k^2-m_l^2+i\epsilon)[(k+k_2)^2+i\epsilon][(k-k_1+\bar{u}p)^2-m_c^2+i\epsilon]}\Big\}. \end{aligned} \tag{26}$$

With the Feynman parameterization, the integration over the loop momentum in the above equation can be

performed, then the coefficient $a_{1\alpha}$ can be expressed as integrals over Feynman parameters

$$a_{1\alpha} = \frac{-i}{16\pi^2}\int_0^1 dx\int_0^{1-x} dy\left\{p_\alpha\left[\frac{1}{a_1}+\frac{u}{a_2} + \frac{\bar{u}}{a_3}\right] - \frac{[y+u(x-1)]k_{1\alpha}+\bar{u}\bar{x}k_{2\alpha}}{a_3} + \frac{-u\bar{x}k_{1\alpha}+(\bar{u}-\bar{u}x-y)k_{2\alpha}}{a_2}\right\}, \quad (27)$$

with

$$\begin{aligned} a_1 &= m_{\eta_c}^2[\bar{u}x+y)(\bar{u}x+y-1)+xu\bar{u}]+xm_c^2 - i\epsilon, \quad (28)\\ a_2 &= [1-2x-2y+(x+y)^2]m_l^2 + xu(\bar{x}\bar{u}-y)m_{\eta_c}^2 - i\epsilon,\\ a_3 &= [1-2x-2y+(x+y)^2]m_l^2 + x\bar{u}(\bar{x}u-y)m_{\eta_c}^2 - i\epsilon. \end{aligned}$$

While the four-point loop integration of the coefficient a_2^σ in eq.(25) can not be reduced, it can be expressed as a 3-fold integral of Feynman parameters

$$a_2^\sigma = \frac{i}{16\pi^2}\int_0^1 dx\int_0^{1-x} dy\int_0^{1-x-y} dz \times\frac{(ux+z-u)k_1^\sigma-(\bar{u}x+y-\bar{u})k_2^\sigma}{a_4^2}, \quad (29)$$

where

$$a_4 = (1-x-y-z)^2 m_l^2 + [xu\bar{u} -(ux+z)(\bar{u}x+y)]m_{\eta_c}^2 + xm_c^2 - i\epsilon. \quad (30)$$

Substitute $a_{1\alpha}$ and a_2^σ in eqs.(27) and (29) into eq.(23), the amplitude A_1 can be reduced to

$$\begin{aligned} A_1 = {} & \frac{Q_c^2e^4}{16\pi^2}f_{\eta_c}\int_0^1 du\phi(u,\mu)\int_0^1 dx\int_0^{1-x} dy m_l \quad (31)\\ & \times\left\{\left[\frac{2}{a_1}+\frac{1+(2u-1)x-y}{a_2}+\frac{1+(1-2u)x-y}{a_3}\right]\cdot\bar{u}(k_1)\gamma_5 v(k_2) + \int_0^{1-x-y} dz\frac{1-x-y-z}{a_4^2}\bar{u}(k_1)\sigma^{\mu\nu}v(k_2)\epsilon_{\mu\nu\rho\sigma}k_1^\rho k_2^\sigma\right\}. \end{aligned}$$

The contribution of Fig.2(b) can be calculated in the same way. It can be finally shown that the contribution of Fig.2(b) A_2 is the same as that of Fig.2(a) if the condition $\phi(u,\mu)=\phi(\bar{u},\mu)$ maintained. This condition is well satisfied by the wave function of η_c.

Then one can obtain the total contribution of the loop-diagrams

$$\begin{aligned} A_l &= A_1 + A_2 \quad (32)\\ &= c_1\bar{u}(k_1)\gamma_5 v(k_2) + c_2\bar{u}(k_1)\sigma^{\mu\nu}v(k_2)\epsilon_{\mu\nu\rho\sigma}k_1^\rho k_2^\sigma, \end{aligned}$$

with

$$c_1 = \frac{Q_c^2e^4}{8\pi^2}f_{\eta_c}m_l\int_0^1 du\phi(u,\mu)\int_0^1 dx\int_0^{1-x} dy \times\left\{\left[\frac{2}{a_1}+\frac{1+(2u-1)x-y}{a_2}+\frac{1+(1-2u)x-y}{a_3}\right],\right. \quad (33)$$

$$c_2 = \frac{Q_c^2e^4}{8\pi^2}f_{\eta_c}m_l\int_0^1 du\phi(u,\mu) \cdot\int_0^1 dx\int_0^{1-x} dy\int_0^{1-x-y} dz\frac{1-x-y-z}{a_4^2}, \quad (34)$$

where $a_1, \sim a_4$ are given in eqs. (28) and (30).

The coefficients c_1 and c_2 are calculated analytically. Before performing the integrations over Feynman parameters, let us make the phase convention in the integrals. The imaginary angles θ's for any imaginary quantities are restricted in the range $-\pi<\theta<\pi$. Then the logarithms appeared during the integration have a cut along the negative real axis. For any two imaginary quantities a and b, the following relations about the logarithm are held [21]

$$\begin{aligned} &\ln(ab) = \ln(a)+\ln(b)+\eta(a,b), \quad (35)\\ &\eta(a,b) = 2\pi i\{\theta(-\mathrm{Im}a)\theta(-\mathrm{Im}b)\theta(\mathrm{Im}ab) - \theta(\mathrm{Im}a)\theta(\mathrm{Im}b)\theta(-\mathrm{Im}ab), \end{aligned}$$

where $\theta(x)$ is the unitstep function.

The analytical results for the Feynman parameter integration are obtained. To express the results conveniently, some quantities are defined as $r_c = m_c^2/m_{\eta_c}^2$, $r_l = m_l^2/m_{\eta_c}^2$, $\alpha = \frac{2r_l}{\sqrt{1-4r_l}}$, and $\beta = \sqrt{1-4r_l}$. The result for c_1 is

$$c_1 = \frac{Q_c^2e^4}{8\pi^2}\frac{f_{\eta_c}m_l}{m_{\eta_c}^2}[f_1(u)+f_2(u)+f_3(u)], \quad (36)$$

where the functions $f_1(u)$, $f_2(u)$ and $f_3(u)$ are

$$\begin{aligned} f_1(u) = {} & \frac{1}{(1-u)u(r_c+(1-u)u)}[2r_c\ln(r_c)+u(1-2r_c-3u+2u^2)\ln(r_c-(1-u)^2-i\epsilon)+(1-u) \quad (37)\\ & \times(u\ln(-r_c+(1-u)^2+i\epsilon)-(2r_c+u-2u^2)\ln(r_c-u^2-i\epsilon)+u\ln(-r_c+u^2+i\epsilon))], \end{aligned}$$

$$
\begin{aligned}
f_2(u) = \Bigg\{ & \frac{\beta}{(1-4r_l)^2}\ \{2(-1+4r_l)[\ln(2)\ln(\frac{1-\beta}{1+\beta}) - \ln(2)\ln(\frac{1+\beta}{1-\beta}) + 2\mathrm{Li}_2(\frac{\beta}{-1+\beta}) - \mathrm{Li}_2(\frac{2\beta}{-1+\beta}) \\
& - 2\mathrm{Li}_2(\frac{\beta}{1+\beta}) + \mathrm{Li}_2(\frac{2\beta}{1+\beta})] + \frac{\beta^2}{-1+\beta^2}[\beta\ln(16) - 2\beta\ln(1+\frac{1}{\beta}) + 2\beta\ln(\frac{1}{\beta}) + 2\beta\ln(\frac{1}{\beta}) \\
& - 2\beta\ln(\frac{1-\beta}{\beta}) - \ln(2)\ln(\frac{1-\beta}{1+\beta}) + \beta^2\ln(2)\ln(\frac{1-\beta}{1+\beta}) + \ln(2)\ln(\frac{1+\beta}{1-\beta}) - \beta^2\ln(2)\ln(\frac{1+\beta}{1-\beta}) \\
& + 2(-1+\beta^2)\mathrm{Li}_2(\frac{\beta}{-1+\beta}) + (1-\beta^2)\mathrm{Li}_2(\frac{2\beta}{-1+\beta}) + 2\mathrm{Li}_2(\frac{\beta}{1+\beta}) - 2\beta^2\mathrm{Li}_2(\frac{\beta}{1+\beta}) - \mathrm{Li}_2(\frac{2\beta}{1+\beta}) \\
& + \beta^2\mathrm{Li}_2(\frac{2\beta}{1+\beta})]\} + \frac{-1+4r_l}{4r_l}\{\frac{1}{1-4r_l}[-t_1\ln(-t_1) + t_1'\ln(-t_1') - t_2\ln(-t_2) + t_2'\ln(-t_2') \\
& + (-1+t_1-\beta)\ln(1-t_1+\beta) + (1-t_1'+\beta)\ln(1-t_1'+\beta) + (-1+t_2-\beta)\ln(1-t_2+\beta) \\
& + (1-t_2'+\beta)\ln(1-t_2'+\beta)] + \frac{4r_l^2}{(1-4r_l)^2}[-\frac{t_1\ln(-t_1)}{\alpha(t_1+\alpha)} + \frac{t_1'\ln(-t_1')}{\alpha(t_1'+\alpha)} - \frac{t_2\ln(-t2)}{\alpha(t2+\alpha)} + \frac{t_2'\ln(-t_2')}{\alpha(t_2'+\alpha)} \\
& - \frac{\ln(\alpha)}{t_1+\alpha} + \frac{\ln(\alpha)}{t_1'+\alpha} - \frac{\ln(\alpha)}{t_2+\alpha} + \frac{\ln(\alpha)}{t_2'+\alpha} - \frac{(1-t_1+\beta)\ln(1-t_1+\beta)}{(t_1+\alpha)(1+\alpha+\beta)} + \frac{(1-t_1'+\beta)\ln(1-t_1'+\beta)}{(t_1'+\alpha)(1+\alpha+\beta} \\
& - \frac{(1-t_2+\beta)\ln(1-t_2+\beta)}{(t_2+\alpha)(1+\alpha+\beta)} + \frac{(1-t_2'+\beta)\ln(1-t_2'+\beta)}{(t_2'+\alpha)(1+\alpha+\beta)} + \frac{\ln(1+\alpha+\beta)}{t_1+\alpha} - \frac{\ln(1+\alpha+\beta)}{t_1'+\alpha} \\
& + \frac{\ln(1+\alpha+\beta)}{t_2+\alpha} - \frac{\ln(1+\alpha+\beta)}{t_2'+\alpha}] + \frac{1}{\beta^3(2r_l+\beta+\beta^2)}\ln(\frac{2r_l+(-1+\sqrt{1-4r_l})u}{-2r_l+(1+\sqrt{1-4r_l})u})[\beta(1+\beta)(4r_l \\
& + \beta+\beta^2) + 4r_l(2r_l+\beta+\beta^2)\ln(2r_l) - 4r_l(2r_l+\beta+\beta^2)\ln(2r_l+\beta+\beta^2)] \\
& - \frac{4r_l}{(1-4r_l)^{3/2}}[\ln(-t_1)\ln(\frac{\alpha}{t_1+\alpha}) - \ln(-t_1')\ln(\frac{\alpha}{t_1'+\alpha}) + \ln(-t_2)\ln(\frac{\alpha}{t_2+\alpha}) \\
& - \ln(-t_2')\ln(\frac{\alpha}{t_2'+\alpha}) - \ln(1-t_1+\beta)\ln(\frac{1+\alpha+\beta}{t_1+\alpha}) + \ln(1-t_1'+\beta)\ln(\frac{1+\alpha+\beta}{t_1'+\alpha}) \\
& - \ln(1-t_2+\beta)\ln(\frac{1+\alpha+\beta}{t_2+\alpha}) + \ln(1-t_2p+\beta)\ln(\frac{1+\alpha+\beta}{t_2'+\alpha}) + \mathrm{Li}_2(\frac{t_1}{t_1+\alpha}) - \mathrm{Li}_2(\frac{t_1'}{t_1'+\alpha}) \\
& + \mathrm{Li}_2(\frac{t_2}{t_2+\alpha}) - \mathrm{Li}_2(\frac{t_2'}{t_2'+\alpha}) - \mathrm{Li}_2(\frac{-1+t_1-\beta}{t_1+\alpha}) + \mathrm{Li}_2(\frac{-1+t_1'-\beta}{t_1'+\alpha}) - \mathrm{Li}_2(\frac{-1+t_2-\beta}{t_2+\alpha}) \\
& + \mathrm{Li}_2(\frac{-1+t_2'-\beta}{t_2'+\alpha})]\} + \frac{i\pi}{2r_l} + \frac{1}{2r_l(r_l+(-1+u)u)}[r_l\ln(-r_l) \\
& + (r_l+(-1+u)u)\ln((1-u)u) - r_l\ln((-1+u)u) - (r_l-u+u^2)\ln(u^2)] \Bigg\},
\end{aligned} \tag{38}
$$

where

$$t_1 = \frac{2r_l(\sqrt{1-4r_l}+u)+2r_l(1-u)}{2r_l-(1+\sqrt{1-4r_l})u} + i\epsilon, \tag{39}$$

$$t_2 = \frac{2r_l(\sqrt{1-4r_l}+u)-2r_l(1-u)}{2r_l-(1+\sqrt{1-4r_l})u} - i\epsilon, \tag{40}$$

$$t_1' = \frac{2r_l(\sqrt{1-4r_l}-u)-2r_l(1-u)}{2r_l+(-1+\sqrt{1-4r_l})u} + i\epsilon, \tag{41}$$

$$t_2' = \frac{2r_l(\sqrt{1-4r_l}-u)+2r_l(1-u)}{2r_l+(-1+\sqrt{1-4r_l})u} - i\epsilon. \tag{42}$$

The function $\mathrm{Li}_2(z)$ is the polylogarithm function, which is defined as $\mathrm{Li}_2(z) = \int_z^0 dt\frac{\ln(1-t)}{t}$. Finally, the third function is

$$f_3(u) = f_2(1-u). \tag{43}$$

The analytical result of the 3-fold Feynman parameter integration in c_2 (eq.(34)) is tedious, it is not presented here. Some formulas used in the procedure of integration are given in Appendix B.

It is not difficult to calculate the contribution of weak interaction, where the virtual Z^0 acts as the intermediate state (Fig.2(c)). The contribution of Fig.2(c) to the amplitude of $\eta_c \to l^+l^-$ is

$$A_{z^0} = \langle l^+l^-|\bar{l}V_{\bar{l}lZ}l\frac{-i}{p^2-m_Z^2+i\epsilon}\bar{c}V_{\bar{c}cZ}c|\eta_c\rangle, \tag{44}$$

where $p = k_1 + k_2$, and $V_{\bar{l}lZ}$ and $V_{\bar{c}cZ}$ are the coupling vertices of $\bar{l}lZ$ and $\bar{c}cZ$, respectively,

$$V_{\bar{l}lZ} = ie\gamma^\mu\left(\frac{-4\sin^2\theta_W+1}{4\sin\theta_W\cos\theta_W} - \frac{1}{4\sin\theta_W\cos\theta_W}\gamma_5\right),$$

$$V_{\bar{c}cZ} = ie\gamma^\mu\left(\frac{8\sin^2\theta_W-3}{12\sin\theta_W\cos\theta_W} + \frac{1}{4\sin\theta_W\cos\theta_W}\gamma_5\right),$$

with θ_W being the Weinberg angle.

Contracting the leptonic final state with the leptonic fields operator and using the definition of the decay constant of η_c, one can finally obtain the amplitude

$$A_{z^0} = \frac{2m_l e^2}{(4\sin\theta_W\cos\theta_W)^2}\frac{f_{\eta_c}}{p^2 - m_Z^2}\bar{u}(k_1)\gamma_5 v(k_2). \quad (45)$$

Then the amplitude of the leptonic decay $\eta_c \to l^+l^-$ is the sum of the loop and tree diagrams

$$A(\eta_c \to l^+l^-) = A_l + A_{Z^0}. \quad (46)$$

The decay width of this decay is

$$\Gamma(\eta_c \to l^+l^-) = \frac{1}{8\pi}|A(\eta_c \to l^+l^-)|^2\frac{|\vec{k}|}{m_{\eta_c}^2}, \quad (47)$$

where $\vec{k}$ is the three-momentum of one of the leptons in the rest frame of η_c. The branching ratio of the decay is defined by

$$Br(\eta_c \to l^+l^-) = \frac{\Gamma(\eta_c \to l^+l^-)}{\Gamma_{\rm tot}}, \quad (48)$$

where $\Gamma_{\rm tot}$ are the total decay width of the η_c meson.

In the numerical calculation, the wave function of η_c is taken to be [18]

$$\phi(u, \mu \sim m_c) = N\ 4u(1-u)\exp\left(-\frac{\beta}{4u(1-u)}\right), \quad (49)$$

where N is the normalization factor, the parameter $\beta = 3.8 \pm 0.7$. The value of the decay constant of η_c is $f_{\eta_c} = 0.346 \pm 0.017$ GeV [18], the mass of c quark $m_c = 1.4$ GeV, the total decay width of η_c is $\Gamma_{\rm tot} = 26.7 \pm 3.0$ MeV [1].

With the parameter inputs and the wave function of η_c given above, the prediction to the branching ratio of $\eta_c \to \gamma\gamma$ is

$$B(\eta_c \to \gamma\gamma) = 2.43^{+0.39}_{-0.34} \times 10^{-4}, \quad (50)$$

where the uncertainty comes from the uncertainty of the total decay width of η_c, the uncertainty of the decay constant f_{η_c} and the uncertainty of the parameter β in the wave function. The main contribution comes from the uncertainties of the total decay width and decay constant of η_c, which are about 10% of the central value, while the error caused by the parameter β is small, which is only about 1%.

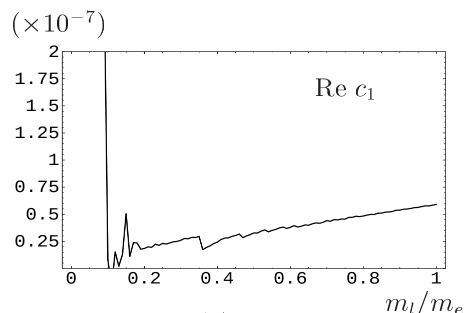
The prediction in eq.(50) is in good agreement with the experimental data

$$B^{\rm exp}(\eta_c \to \gamma\gamma) = 2.4^{+1.1}_{-0.9} \times 10^{-4}. \quad (51)$$

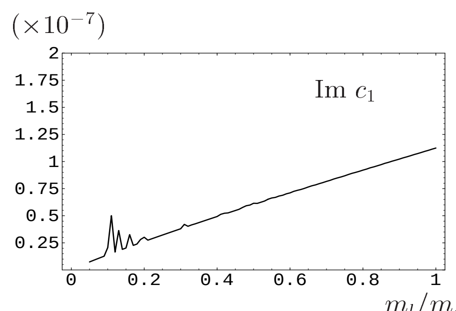
Next I will go on to the numerical discussion on $\eta_c \to l^+l^-$.

The loop integral is infrared divergent if the lepton mass is zero. This can be shown by taking the asymptotic limit of $r_l \to 0$ for the analytical results of loop-integral-functions $f_1(u)$, $f_2(u)$ and $f_3(u)$ in eqs. (37), (38) and (43). The most singular term behaves like $\frac{1}{r_l}\ln(r_l)$. Considering the decay amplitude in eq. (31) is proportional to $m_l \sim \sqrt{r_l}$ due to the helicity suppression, the most singular term in the decay amplitude behaves like $\frac{1}{\sqrt{r_l}}\ln(r_l)$. The curves for the coefficient c_1 as the lepton mass approaching zero are shown in Fig.3. Only when the lepton mass $m_l < 0.2m_e$, here m_e is the mass of electron, does the numerical result begin to be severely affected by the infrared divergence. As the lepton mass not less than $0.2m_e$, the numerical value is reliable. The situation for c_2 is similar to c_1. Therefore, for the cases of $m_l = m_e$ and $m_l = m_\mu$ (m_μ the muon mass), the loop integral is not affected by the infrared divergence.

The values of c_1 and c_2 are presented in Table I.



(a)



(b)

FIG. 3: Behavior of the coefficient c_1 as $m_l \to 0$. (a) the real part of c_1; (b) the imaginary part of c_1. The horizontal axis is for the ratio of m_l/m_e.

TABLE I: The coefficient c_1 and c_2 in the decay amplitudes of $\eta_c \to e^+e^-$ and $\eta_c \to \mu^+\mu^-$.

	c_1	c_2
$\eta_c \to e^+e^-$	$5.90 \times 10^{-8} + 1.12 \times 10^{-7} i$	$-2.75 \times 10^{-8} - 1.43 \times 10^{-9} i$
$\eta_c \to \mu^+\mu^-$	$3.20 \times 10^{-6} + 1.02 \times 10^{-5} i$	$-5.53 \times 10^{-7} + 5.36 \times 10^{-6} i$

The branching ratios are given in Table II. The leptonic decays of η_c are dominated by the two-photon loop diagrams, the contribution of Z^0 tree diagram modifies the decay rates by less than 1%.

TABLE II: The branching ratios of $\eta_c \to e^+e^-$ and $\eta_c \to \mu^+\mu^-$. The column "Only loop" means the contribution of only loop diagram, without the contribution of Z^0 tree diagram, while "Loop+Z^0" denotes the contribution of both loop and Z^0 tree diagrams.

	Only loop	Loop+Z^0
$\eta_c \to e^+e^-$	$4.77^{+0.77}_{-0.66} \times 10^{-13}$	$4.74^{+0.77}_{-0.66} \times 10^{-13}$
$\eta_c \to \mu^+\mu^-$	$6.41^{+1.04}_{-0.89} \times 10^{-9}$	$6.39^{+1.03}_{-0.89} \times 10^{-9}$

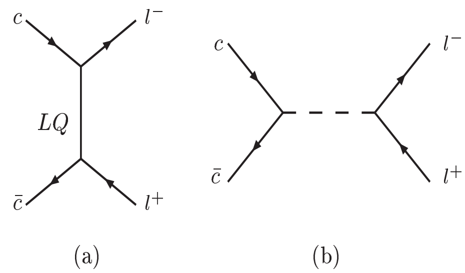


FIG. 4: Diagrams for $\eta_c \to l^+l^-$ in physics beyond the standard model. (a) leptoquark contribution, and (b) light pseudoscalar Higgs contribution.

The branching ratios of $\eta_c \to l^+l^-$ are very tiny within the frame work of the standard model. Charm physics including leptonic, semileptonic and hadronic charm decays will be studied at BESIII [22, 23]. About 10×10^9 J/ψ events will be accumulated with one year designed luminosity. Considering the branching ratio of $J/\psi \to \gamma\eta_c$ being $(1.3 \pm 0.4)\%$ [1], about 1.3×10^8 η_c mesons can be produced through the radiative decays of $J/\psi \to \gamma\eta_c$. Assuming the detection efficiency is about 20%, then the sensitivity of the measurement of $\eta_c \to l^+l^-$ can reach 3.8×10^{-8}. The standard model prediction to the leptonic decay of $\eta_c \to l^+l^-$ is just below the sensitivity of BESIII. However, the low decay rate within the standard model makes the leptonic decay of $\eta_c \to l^+l^-$ sensitive to physics beyond the standard model, such as the existence of leptoquark bosons, Fig. 4(a), or the light pseudoscalar Higgs boson, Fig. 4(b). Especially when the mass of the light pseudoscalar Higgs lies near the mass of η_c meson, Fig. 4(b) can significantly enhance the leptonic decay rate of η_c. Measurement of $\eta_c \to l^+l^-$ decay, especially for $\eta_c \to \mu^+\mu^-$ channel, can give information of physics beyond the standard model.

In summary, the radiative and leptonic decays of $\eta_c \to \gamma\gamma$ and $\eta_c \to l^+l^-$ have been studied consistently within the framework of the standard model. The theoretical prediction of the decay rate of $\eta_c \to \gamma\gamma$ is in well agreement with the experimental measurement. For the leptonic decays of $\eta_c \to \mu^+\mu^-$ and e^+e^-, within the framework of the standard model, the loop diagrams of electromagnetic interaction dominate, the tree diagram of weak interaction involving z^0 propagator can only modifies the decay rates by less than 1%. The decay rates of $\eta_c \to \mu^+\mu^-$ and e^+e^- are tiny, which makes them sensitive to some new physics beyond the standard model. Measurement of these decay rates may shed light on the existence of new physics.

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Appendix A

The wave function of a meson M composed of a quark-antiquark pair $\bar{q}_1 q_2$ can be defined through the matrix element $\langle 0|\bar{q}_1(x)_\alpha q_2(y)_\beta|M\rangle$, where summation over color degrees of freedom and a gauge invariant factor $P\exp\{i\int dz^\mu T^a A^a_\mu(z)\}$ between $\bar{q}_1(x)$ and $q_2(y)$ are indicated.

Using the equality of Fietz transformation

$$\delta_{ik}\delta_{lj} = \frac{1}{4}\delta_{ij}\delta_{lk} + \frac{1}{4}(\gamma_5)_{ij}(\gamma_5)_{lk} + \frac{1}{4}(\gamma_\mu)_{ij}(\gamma^\mu)_{lk} - \frac{1}{4}(\gamma_\mu\gamma_5)_{ij}(\gamma^\mu\gamma_5)_{lk} + \frac{1}{8}(\sigma_{\mu\nu}\gamma_5)_{ij}(\sigma^{\mu\nu}\gamma_5)_{lk}, \tag{52}$$

the matrix element can be written as

$$\begin{aligned}
&\langle 0|\bar{q}_1(x)_\alpha q_2(y)_\beta|M\rangle \qquad (53)\\
=\ &\langle 0|\bar{q}_1(x)_\rho q_2(y)_\sigma|M\rangle\delta_{\rho\alpha}\delta_{\beta\sigma}\\
=\ &\langle 0|\bar{q}_1(x)_\rho q_2(y)_\sigma|M\rangle(\frac{1}{4}\delta_{\rho\sigma}\delta_{\beta\alpha}+\frac{1}{4}(\gamma_5)_{\rho\sigma}(\gamma_5)_{\beta\alpha}\\
&+\frac{1}{4}(\gamma_\mu)_{\rho\sigma}(\gamma^\mu)_{\beta\alpha}-\frac{1}{4}(\gamma_\mu\gamma_5)_{\rho\sigma}(\gamma^\mu\gamma_5)_{\beta\alpha}\\
&+\frac{1}{8}(\sigma_{\mu\nu}\gamma_5)_{\rho\sigma}(\sigma^{\mu\nu}\gamma_5)_{\beta\alpha})\\
=\ &\frac{1}{4}\langle 0|\bar{q}_1(x)q_2(y)|M\rangle\delta_{\beta\alpha}+\frac{1}{4}\langle 0|\bar{q}_1(x)\gamma_5q_2(y)|M\rangle(\gamma_5)_{\beta\alpha}\\
&+\frac{1}{4}\langle 0|\bar{q}_1(x)\gamma_\mu q_2(y)|M\rangle(\gamma^\mu)_{\beta\alpha}\\
&-\frac{1}{4}\langle 0|\bar{q}_1(x)\gamma_\mu\gamma_5q_2(y)|M\rangle(\gamma^\mu\gamma_5)_{\beta\alpha}\\
&+\frac{1}{8}\langle 0|\bar{q}_1(x)\sigma_{\mu\nu}\gamma_5q_2(y)|M\rangle(\sigma^{\mu\nu}\gamma_5)_{\beta\alpha}.
\end{aligned}$$

For a pseudoscalar meson M, the matrix elements of the scalar and vector current are all zero. For the nonzero matrix element the wave function can be defined as [20]

$$\langle 0|\bar{q}_1(x)\gamma_\mu\gamma_5q_2(y)|M\rangle = if_Mp_\mu\int_0^1 due^{-i(up\cdot x+\bar{u}p\cdot y)}\phi(u,\mu), \qquad (54)$$

$$\langle 0|\bar{q}_1(x)i\gamma_5q_2(y)|M\rangle = \frac{f_Mm_M^2}{m_1+m_2}\int_0^1 due^{-i(up\cdot x+\bar{u}p\cdot y)}\phi_p(u,\mu), \qquad (55)$$

$$\begin{aligned}
\langle 0|\bar{q}_1(x)\sigma_{\mu\nu}\gamma_5q_2(y)|M\rangle\ =\ &i\frac{f_Mm_M^2}{m_1+m_2}(p_\mu z_\nu-p_\nu z_\mu) \qquad (56)\\
&\cdot\int_0^1 due^{-i(up\cdot x+\bar{u}p\cdot y)}\frac{\phi_\sigma(u,\mu)}{6},
\end{aligned}$$

where $z=y-x$, μ is the energy scalar where the perturbative and non-perturbative dynamics of QCD can be factorized, f_M the decay constant, p_μ the momentum of the meson M, and m_M, m_1 and m_2 are the masses of the meson M, quarks q_1 and q_2, respectively. The wave function $\phi(u,\mu)$ is of twist-2, and $\phi_p(u,\mu)$ and $\phi_\sigma(u,\mu)$ are of twist-3.

Then the matrix element $\langle 0|\bar{q}_1(x)_\alpha q_2(y)_\beta|M\rangle$ becomes

$$\begin{aligned}
&\langle 0|\bar{q}_1(x)_\alpha q_2(y)_\beta|M\rangle \qquad (57)\\
&=-\frac{i}{4}f_M\int_0^1 due^{-i(up\cdot x+\bar{u}p\cdot y)}[\not{p}\gamma_5\phi(u,\mu)\\
&+\frac{m_M^2}{m_1+m_2}\gamma_5\phi_p(u,\mu)-\frac{1}{2}\frac{m_M^2}{m_1+m_2}(p_\mu z_\nu\\
&-p_\nu z_\mu)\sigma_{\mu\nu}\frac{\phi_\sigma(u,\mu)}{6}\gamma_5]_{\beta\alpha}.
\end{aligned}$$

For the decay processes of η_c considered in this work, only the leading-twist wave function is considered. The contributions of the wave functions of twist-3 are suppressed due to their spin structure. Therefore they are safely dropped.

Appendix B

Some formulas used in the calculation of the 3-fold Feynman parameter integration are

$$\begin{aligned}
\int dx\frac{1}{ax^2+bx+c} &= \frac{1}{\sqrt{b^2-4ac}}[\ \ln(2ax+b \qquad (58)\\
&-\sqrt{b^2-4ac}\ ]-\ln(2ax+b+\sqrt{b^2-4ac}\ ],
\end{aligned}$$

$$\begin{aligned}
\int dx\frac{x}{(ax^2+bx+c)^2} &= \frac{bx+2c}{(b^2-4ac)(ax^2+bx+c)} \qquad (59)\\
&+\frac{b}{b^2-4ac}\int dx\frac{1}{ax^2+bx+c}.
\end{aligned}$$

For the integration of the type

$$\int dy\frac{n_1y+n_2}{(a_1y^2+b_1y+c_1)(a_2y^2+b_2y+c_2)}, \qquad (60)$$

the integrand can be decomposed as

$$\begin{aligned}
&\frac{n_1y+n_2}{(a_1y^2+b_1y+c_1)(a_2y^2+b_2y+c_2)} \qquad (61)\\
&=\frac{k_1y+l_1}{a_1y^2+b_1y+c_1}+\frac{k_2y+l_2}{a_2y^2+b_2y+c_2},
\end{aligned}$$

where the coefficients $k_{1,2}$ and $l_{1,2}$ can be solved within the above equation. Then the integration of eq. (60) can be reduced into a simpler form.

For the integral like

$$\int dx\frac{kx+l}{ax^2+bx+c}\frac{1}{\sqrt{ex+f}}\ln(gx+b\pm\sqrt{ex+f}), \quad (62)$$

the integration can be performed by making the variable transformation $t=\sqrt{ex+f}$. While for

$$\begin{aligned}
&\int dx\frac{kx+l}{a_1x^2+b_1x+c_1}\frac{1}{\sqrt{a_2x^2+b_2x+c_2}} \qquad (63)\\
&\times\ln(gx+b\pm\sqrt{a_2x^2+b_2x+c_2}),
\end{aligned}$$

with $a_2>0$, the variable transformation shall be $\sqrt{a_2x^2+b_2x+c_2}=t-\sqrt{a_2}x$.

$$\begin{aligned}
\int dt\frac{\ln(t-a)}{t-b}\ =\ &\ln(t-a)\ln\left(\frac{t-a}{a-b}+1\right) \qquad (64)\\
&+\mathrm{Li}_2\left(-\frac{t-a}{a-b}\right),
\end{aligned}$$

where $\mathrm{Li}_2(z)$ is the polylogarithm function, which is defined as $\mathrm{Li}_2(z)=\int_z^0\frac{\ln(1-t)}{t}$.

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